Abstract
Waveguide theory is suggested for the analysis of UHF radiation in urban microcellular environments, where both the transmitter and the receiver are below rooftop level. The delay profile is calculated from the steady state power distribution among the modes of the waveguide. The delay profile is very asymmetrical because most of the power is contained in the low order (faster) modes, and because the differences between the group velocities of the modes tend to be smaller for the low order modes.

1 Introduction
Microcellular propagation is customarily analyzed using ray tracing, with various accommodations for diffraction and penetration. Simulation tools based on ray tracing are used to predict coverage and other characteristics of UHF propagation, especially in built up areas (see [1, 5, 3, 2, 6, 12] among many other examples). We suggest an alternative treatment of microcellular urban propagation, based on the analysis of streets as waveguides. The modal analysis offers some interesting intuitions on the propagation process, which are not readily obvious from ray analysis. Specifically, we investigate in this paper the implications of the modal theory on the delay profile in microcellular urban environments.

2 The Modal Theory of Streets
Our propagation model is based on the analysis of streets as two dimensional (slab) waveguides, with a ground plane. Both the transmitter and the receiver are assumed below rooftop level, so the walls of the waveguide are approximated as infinitely high. The street waveguide is heavily multimoded because typical wavelengths in the UHF band are at least an order of magnitude smaller than the width of a typical street.

An important concept in the waveguide analysis of streets is mode coupling. This is caused by the perturbations of the walls of the waveguide, or the facades of buildings on both sides of the street. Actual streets differ from our model as they contain objects such as cars and people, which further perturb the modal structure.

The average mode coupling can be predicted from the electrical properties of the waveguide and the statistics of the wall perturbations. The theory was developed for optical waveguides [7, 8] and extended in [10] to the street canyon case. The coupling model is given by a linear differential equation which describes the relationships among the propagating modes:

\[
\frac{\partial P}{\partial z} = \Gamma P
\]  

where \(P(z)\) is a vector containing the power in each mode and \(z\) is parallel to the waveguide. The matrix \(\Gamma\) contains the loss coefficients of all the modes on the diagonal, and the average coupling coefficients between the modes in the off–diagonal elements. The coupling coefficients depend on the electrical properties of the waveguide walls, the geometry of the waveguide and the statistical properties of the wall perturbations, see appendix A for more details. The linear coupling model holds under some simplifying assumptions on the coupling process, in particular a perturbation assumption which takes into account slow coupling. This assumption may be questionable in microcellular environments; we use the linear coupling model as a first order approximation.

The coupled power equation yields a steady state distribution of power among the propagating modes, which is attained at sufficient distance from the transmitter and
major discontinuities of the waveguide. The steady state distribution is determined from the first eigen-vector of the matrix $\Gamma$. Measurements indicate that the convergence distance of the power distribution to the steady state one is on the order of tens to hundreds of meters [10]. The typical steady state distribution of power has most of the power in the low order modes, which are less lossy than the higher order modes.

The effect of the ground is added by considering a single reflection off the ground, which adds coherently to the direct modal path. For reasonable distances from the transmitter this amounts to a 3 dB increase in power levels.

The group velocity is the speed of power carried by each mode [11], for the $n^{th}$ mode in a hollow slab waveguide it is given by

$$v_n = c \cos(\theta_n)$$

where $c$ is the speed of light in air and $\theta_n$ is the angle between the wave vector of the $n^{th}$ mode and the direction of the waveguide (figure 1). The velocity of information transfer equals the group velocity in a single mode waveguide. In multi-moded waveguides with no mode coupling, a single input pulse is split into a number of pulses that propagate down the waveguide, each output pulse carried by a different mode and travels at its group velocity. In a waveguide with mode coupling the situation is more complicated. Energy is exchanged among the modes, so the response of the waveguide to an input pulse is a distorted and longer pulse instead of the comb–like response of the waveguide with no mode coupling. Mode coupling decreases the dispersion of waveguides, because it forces some power to switch between fast modes and slow modes [9].

The delay spread calculated for a waveguide without mode coupling is an upper bound on the delay spread in similar waveguides with mode coupling. Consider a waveguide of length $L$ made of a similar material and with similar dimensions to the street waveguide but without mode coupling. The distribution of power among the modes of the waveguide is taken from the steady state distribution calculated for the street waveguide. The power delay profile in this idealized waveguide is given by:

$$P(t) = \sum_{n=1}^{N} P_n \delta(t - \tau_n)$$

where $P_n$ is the power carried by the $n^{th}$ mode and $\tau_n$ is the delay, given by $\tau_n = L/v_n$.

3 The Delay Profile

In order to better understand the propagating modes, we look at their wave vectors $k_n$, and note that the low order modes are clustered in the small angles (figure 2). The difference between their group velocities is smaller than the difference between the group velocities of the high order modes. Therefore, the delays of the low order modes are spread with small differences between them, compared to the differences of delays between the high order mode. This clustering of the power at low delays creates a non-uniform delay profile (figure 3), where the profile is more dense at the low delays. The steady state distribution of power used in figure 3 was calculated for a 10 m wide street, with the relative dielectric constant of the walls set at $\epsilon_r = 10$, the conductivity set at $\sigma = 0.4$ S/m, the variance of the wall perturbation 0.04 m$^2$ and the correlation length of the wall perturbation set at 5 m. The carrier frequency is 2.6 GHz.

Another reason for the importance of the low delays is the distribution of power among the propagating modes. The steady state distribution of power among the modes contains almost all the power in the low order modes, which propagate with lower delays. This concentration of power, together with the clustering at the low delays,
Figure 2: The modes of a hollow slab waveguide, described in an angular diagram. The cross–street component of the wave vector increases at more or less fixed steps, while the parallel components of the low order modes are clustered.

Figure 3: Delay profile of a multi-moded waveguide 300 m long with no mode coupling. The arrival time and power of each mode is indicated by a vertical line. Power distribution among the modes is the steady state distribution.

Make very asymmetrical delay profiles with most of the power in the low delays.

For a transmitter located below roof level, power is spread over an urban area mainly via the streets. The major propagation paths flow in the streets and around corners. As power flows from one (‘main’) street into another intersecting it (the ‘cross’ street), the distribution of power among the modes in the cross street is set by the coupling mechanism at the intersection. The power distribution then sets to the steady state distribution at sufficient distances from the junction. The reference point for the delay spread in the cross street is the junction.

One conclusion from the above argument is that the shape of the delay profile is only mildly dependent on the distance from the transmitter. Rather, it depends strongly on the distance from the intersection where significant coupling takes place. The distance from the transmitter affects the delay profile by shifting it without a major change of its shape.

We compare our calculated delay profiles to the empirical model suggested by Ichitsubo et al. [4] for line of sight (LOS) in streets at 2.6 GHz. We consider the LOS case the most appropriate for comparison, because our model does not include propagation paths over the roofs of buildings, and each street intersection where power couples from one street to another effectively resets the shape of the delay profile.

The delay profile from figure 3 is plotted again in figure 4, where the graph represents the received power over time. Overlaying it is a delay profile calculated using Ichitsubo’s empirical model [4] for LOS microcellular streets, with the transmitter and receiver 300 m apart. The empirical model is a power function of time, and good similarity is evident between the two profiles.

The parameters used for the theoretical calculation were chosen for best match with the empirical model. It appears that these parameters describe the average electrical and geometrical properties of the environment of measurement (in the metropolitan area of Tokyo), but this was not validated otherwise. The agreement between the two models is good for transmitter–receiver separations in the range of 200–400 m, where 400 m is the range of validity of the empirical model [4]. For smaller transmitter–receiver separations, the assumption that the power distribution among the modes is similar to the steady state one is not valid, and the calculation of the delay profile based
4 Conclusion

The implications of the modal analysis of streets on the delay profile were presented. Modal analysis was suggested as an alternative to ray tracing in microcellular urban environments, where both the transmitter and receiver are below rooftop level. The coupling among the waveguide modes and their losses induce a steady state power distribution. This in turn translates into an exponential-like delay profile. A typical delay profile calculated using the waveguide theory compares well with an empirical model.

A The Coupling Matrix

The coupling matrix $\Gamma$ in (1) contains the average coupling and loss of the propagating modes. The expressions below are similar to those presented by Marcuse in [8], with an adaptation for hollow dielectric waveguides; see [10] for more details.

The off-diagonal element $\Gamma_{n,m}$ contains the power coupling coefficient from mode $m$ to mode $n$

$$\Gamma_{n,m} = \sqrt{\pi \sigma^2} D K_n e^{-\left[\Phi_{\beta_m} - \beta_n\right]^2} n = 1, 2, \ldots N$$

where $N$ is the number of propagating modes and

$$K_{n,m} = |\epsilon_w - 1|^2 \frac{k^4}{4d^2 \beta_n \beta_m} \left\{ \begin{array}{c} \cos^2 u_n \\ \sin^2 u_n \end{array} \right\} \left\{ \begin{array}{c} \cos^2 u_m \\ \sin^2 u_m \end{array} \right\}$$

$2d$ is the width of the street, $\sigma^2$ is the variance of the wall perturbations which are stationary by assumption, and $D$ is the correlation length of the wall perturbations. $\epsilon_w$ is the relative complex dielectric constant of the walls, that contains the effects of conductivity. $\beta_n = k \cos(\theta_n)$ is the $z$ component of the wave vector and $u_n = dk \sin(\theta_n)$ is the normalized cross street component. $k = w/c$, where $w$ is the radial frequency and $c$ is the speed of light. The $\cos$ variation is used for the symmetrical modes, and the $\sin$ is used for the asymmetrical modes. The model is two dimensional, so the wall perturbations do not vary in the vertical direction. The modes in the waveguide are divided into two polarizations, namely TE modes with a vertical electric field and TM modes with a vertical magnetic field. The model does not account for cross-polarization coupling, but we introduce it by using (4) for every mode combination, of similar or different polarizations.

The diagonal element $\Gamma_{n,n}$ contains the loss factor of the $n^{th}$ mode

$$\Gamma_{n,n} = -\alpha_n - \sum_{m=1, m \neq n}^N \Gamma_{n,m}$$

where $\alpha_n$ is the modal loss factor for a waveguide without mode coupling.

References


