Speed Estimation in Wireless Systems Using Wavelets

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Abstract—This paper presents a new technique for estimating the speed of a mobile station in a wireless system. The mobile speed maps the characteristic spatial scale of the received signal into a characteristic temporal scale. The continuous wavelet transform tracks changes in the temporal scale to estimate the mobile speed as a function of time. This technique requires neither knowledge of the average received power of the nonstationary signal nor adaptation of a temporal observation window, in contrast to other speed estimators given in the literature. Simulation results indicate the ability to track a variable speed profile.

I. INTRODUCTION

A significant characteristic of wireless systems is the signal variation caused by the movement of the mobile stations. Estimating the speed of a mobile station is desirable for several reasons. One application of speed estimation is to determine the duration of a temporal window over which the received signal is averaged to mitigate signal variation. Mobile speed can also be used to determine whether a mobile station which requests access to the wireless system should be assigned to a microcell (for low mobile speeds) or to an umbrella macrocell (for high mobile speeds). System control algorithms such as handoff algorithms can also benefit from knowledge of mobile speed.

In many environments, a direct path is not present between the base station and the mobile station. The received signal consists of a sum of waves which have been reflected by objects such as mountains, trees, and buildings. The sum of many waves at the receiver gives rise to small scale spatial variation of the received signal (on the order of a wavelength). In situations where there is no dominant path between base station and mobile station, the small scale spatial variation is called Rayleigh fading. The received signal is nonstationary for distances on the order of building sizes since the mean of the small scale variation changes considerably. This large scale variation of the mean is known as shadowing. The mean of the shadowing also decreases as the distance between base station and mobile station increases.

Estimates of the mobile speed can be obtained by using the statistics of the received signal. For example, level crossing rates [1] or the autocovariance [2] of the received envelope have been used to estimate speed. Speed estimates have also been obtained by estimating the maximum Doppler frequency using eigenspace methods [3], spectrum estimation methods [4], and the squared deviations of the logarithmically compressed envelope [5]. Another method of velocity estimation requires knowledge of the average signal strength for all locations within a region of interest and uses a technique similar to the multidimensional scaling (MDS) method of statistical data analysis [6]. All of the above techniques require estimates of the signal power, and some methods also require the signal autocorrelation. A difficulty in obtaining such estimates is the nonstationary nature of the received signal. An appropriate window which depends on the unknown mobile speed must be chosen to estimate the required quantities. Furthermore, the literature mentioned above has considered only the problem of a constant, unknown mobile speed. For variable speeds, the duration of the observation window must be constantly adapted, and the rate of adaptation will be critical to the performance of the speed estimator. In particular, errors in the speed estimates could propagate due to suboptimal observation windows.

This paper presents a new method of speed estimation using wavelet analysis. Wavelet methods are useful for this problem since the wavelet transform at different scales corresponds to a variety of window lengths. This choice of windows eliminates the necessity of adapting the duration of a single temporal observation window. The method presented here utilizes the fact that the small scale spatial variation of the received envelope depends only on the positions of the mobile and base stations. This spatial variation has a characteristic scale which is on the order of a carrier wavelength. The temporal variation of the received envelope is then a consequence of mapping the spatial variation through the mobile speed. By tracking the characteristic temporal scale of the variations, an estimate of the speed is obtained as a function of time. In Section II, a wireless propagation model is presented. Section III presents a method for speed estimation using the continuous wavelet transform (CWT). Section IV addresses the selection of parameters used in the speed estimate, compares the performance with other estimators for constant mobile speed, and gives results of simulations using a realistic, variable speed profile. Conclusions are given in Section V.

II. WIRELESS PROPAGATION MODEL

The propagation model discussed here takes into account three effects which are present in many wireless environments: correlated Rayleigh fading, correlated lognormal shadowing, and a distance-dependent trend. The received bandpass signal consists of a sum of contributions from several paths. Let $T_r(x)$ and $T_i(x)$ denote the in-phase and quadrature components of the received signal at position $x$. By the Central Limit Theorem, $T_r(x)$ and $T_i(x)$ are independent, identically distributed, wide-sense stationary, zero-mean Gaussian random processes in a small neighborhood of $x$. The size of the neighborhood of...
where $\sigma_y^2(x, y)$ and $d_0(x, y)$ are the variance and correlation length of $L(x, y)$. The power spectrum $S_L(v, x_B)$ of $L(x, y)$ is given by

$$S_L(v, x_B) = \frac{2d_0(x_B)\sigma_y^2(x_B)}{1 + (2\pi v d_0(x_B))^2},$$

where $v$ denotes spatial frequency. Let $v_{\text{max}}$ denote the maximum spatial frequency of $S_L(v, x_B)$ that is taken into account, and let $D$ be the magnitude of the total displacement of the mobile within a time period of interest. A model for $L(x, y)$ can be shown to be

$$L(x, y) = \sum_{j=-j}^{L-1} \left\{ \frac{2\pi}{CD} S_L \left( \frac{(j + 1/2)}{D}, x_B \right) \right\}^{1/2} \cos \left[ \frac{2\pi(x, y)}{D} (j + 1/2) + \beta_j(x, x_B) \right],$$

where

$$C = \frac{1}{D \sigma_y^2(x_B)} \sum_{j=-j}^{L-1} S_L \left( \frac{(j + 1/2)}{D}, x_B \right),$$

$$v_{\text{max}} = J/D,$$

and $\beta_j(x, x_B)$ are independent, identically distributed uniformly on $[0, 2\pi]$. The received envelope is Rayleigh distributed. The received signal envelope $r(x)$ and its logarithm $s(x)$ are then given by

$$r(x) = \sqrt{T^2(x) + T^2(x)},$$

$$s(x) = 20 \log_{10}[r(x)].$$

The model presented here is used in the speed estimation method described in Section III.

### III. SPEED ESTIMATION USING CWT

The speed estimation technique presented here utilizes the characteristic spatial scale due to correlated Rayleigh fading of the received signal. In this analysis, the signal of interest is the logarithm of the envelope. Let the curve $\Gamma$ traversed by the mobile station be parametrized by the scalar position variable $x$ and let $g(x)$ represent the received signal (i.e. the logarithm of the envelope) as a function of $x$. Fig. 1 plots a typical signal trace as a function of position (measured in wavelengths). This plot shows that the local minima of the signal occur with a variation of a fraction of a wavelength. Suppose that the mobile travels along $\Gamma$ with a speed $v(t)$ at time $t$. The received signal $f(t)$ as a function of $t$ is then

$$f(t) = g \left( \int_0^t v(t') dt' \right),$$

where we have taken the time origin to correspond to $x = 0$. This model accounts for Rayleigh fading in the absence of additive noise. Let the mean separation in distance between the
local minima of \( g(x) \) be \( k\lambda \), where \( 0 < k < 1 \). If the mean separation in a neighborhood of time \( t \) between the local minima of \( f(t) \) is \( \Delta T(t) \), an estimate \( \hat{\nu}(t) \) for the speed can be obtained as

\[
\hat{\nu}(t) = \frac{k\lambda}{\Delta T(t)}.
\]

A method to estimate \( \Delta T(t) \) using the continuous wavelet transform is described in the following.

The continuous wavelet transform (CWT) of a function \( h(t) \in L_2(\mathbb{R}) \) is given by

\[
CWT_h(a,b) \equiv \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi\left(\frac{t-b}{a}\right) h(t) \, dt,
\]

where \( a \in \mathbb{R}^+ \) denotes “scale” and \( b \in \mathbb{R} \) denotes “shift”. The “mother wavelet” \( \psi(t) \in L_2(\mathbb{R}) \) is taken to be real and satisfies the admissibility condition

\[
\int_{-\infty}^{\infty} |\psi(\omega)|^2 \, d\omega = \int_{-\infty}^{0} \frac{|\psi(\omega)|^2}{|\omega|} \, d\omega < \infty,
\]

where \( \psi(\omega) \) is the Fourier transform of \( \psi(t) \).

The CWT has the important property of characterizing singularities of the signal \( h(t) \) \([10, 11, 12]\). Let all derivatives of \( h(t) \) up to order \( n \) exist and be of bounded variation. If the \( n \)-th derivative of \( h(t) \) is discontinuous at \( t = t_0 \), then for a constant \( c_n \), \( CWT_n(h_0, t_0) \approx c_n a^{n+1/2} \) as \( a \to 0 \). As shown in Fig. 1, many of the local minima of \( g(x) \) correspond to points of discontinuity in the first derivative. Thus, the CWT behaves as \( a^{3/2} \) for small \( a \) near the local minima of \( g(x) \) with discontinuous derivative.

The CWT is applied to the received signal \( f(t) \) in order to detect the points of discontinuity in the first derivative of \( f(t) \). In order to obtain speed estimates within an acceptable delay for real-time implementation, the CWT of \( f(t) \) is taken over a suitable temporal window of observation. The CWT is then computed for a set of scales \( a = 2^{jM/m^j}, \quad j = J_{\min}, J_{\min} + 1, \ldots, J_{\max}, \quad m = 0, \ldots, M - 1 \) where \( J_{\min}, J_{\max} \) and \( M \) are integers. In order to compare the CWT with a significance threshold which is independent of mobile speed, the CWT is normalized as follows. Let \( f_0(t) \) be the received signal for a constant mobile speed \( v_0 \) and \( f_1(t) = f_0(t/A) \) be the received signal for a constant mobile speed \( v_1 = v_0/A \). Definition (14) relates the CWT's of \( f_0(t) \) and \( f_1(t) \):

\[
CWT_f(a,b) = \sqrt{A} \cdot CWT_{f_0}(a/A, b/A).
\]

Thus, the CWT of \( f(t) \) at each scale \( a \) is normalized by \( 1/\sqrt{a} \) before further processing.

One method to detect the points of discontinuity in the first derivative of \( f(t) \) is to identify the wavelet transform modulus maxima \([10]\). The number of wavelet transform modulus maxima at fine scales which are associated with a singularity of \( f(t) \) depends upon the number of local extrema of the analyzing wavelet \( \psi(t) \). The number of local extrema of the wavelet is at least one plus the number of vanishing moments of the wavelet. Furthermore, the analyzing wavelet must have at least two vanishing moments in order to characterize points of discontinuous derivative of \( f(t) \). In order to reduce the number of extraneous wavelet transform modulus maxima which are associated with a singularity of \( f(t) \), the following method is adopted. For wavelets \( \psi(t) \) which satisfy \( \sup |\psi(t)| \geq -\inf |\psi(t)| \), the negative local minima at each scale of the normalized CWT are identified; otherwise, the positive local maxima at each scale are identified. The values which are identified by the method described above will be referred to as the signed local extrema of the normalized CWT. The signed local extrema with absolute value less than a significance threshold \( \tau \) are discarded. The scale which has the highest number of significant signed local extrema is identified as the scale of interest and is denoted as \( a_0 \).

Since most of the local minima of \( f(t) \) correspond to points of discontinuity in the first derivative, the locations in time of the signed local extrema of the normalized CWT correspond to most of the locations of the local minima of the signal \( f(t) \). To compensate for the small fraction of local minima of \( f(t) \) with continuous derivative, the significance threshold \( \tau \) is determined empirically such that the number of local minima of \( f(t) \) which are detected by the CWT is equal to the total number of local minima of \( f(t) \). We now consider \( (N + 1) \) significant signed local extrema. For a fixed \( N \), these signed local extrema correspond to a time interval \( t \in [t_0,t_1] \) which depends on the mobile speed. Let \( \Delta t_n \) denote the time between the \((n - 1)\)-th and \( n\)-th signed local extrema. The estimate (13) for \( \nu(t) \) at time \( (t_0 + t_1)/2 \) becomes

\[
\hat{\nu}\left(\frac{t_0 + t_1}{2}\right) = \frac{k\lambda}{\sum_{n=1}^{N} \Delta t_n}.
\]

The next speed estimate sample, \( \hat{\nu}(t_0' + t_1')/2 \), is obtained by averaging over a time interval \([t_0',t_1']/2 \) which contains a new
signed local extremum and the $N$ most recent signed local extrema from the previous time interval. The speed estimates obtained in this manner are smoothed by a moving average. The following section describes the selection of parameters used in the above speed estimate together with simulations which apply the estimation technique.

IV. PARAMETER SELECTION AND SIMULATION RESULTS

This section addresses the determination of the mean distance between local minima of the signal ($k_\lambda$), the selection of the significance threshold ($\tau$), and number ($N$) of interarrival times of the signed local extrema to be used in the speed estimate. The choice of the maximum scale ($a_{\text{max}}$), the duration ($T_{\text{obs}}$) of the observation window over which the CWT is taken, and associated boundary effects are also described. The observation window is required to obtain speed estimates within an acceptable delay for real-time implementation. Finally, an application of the speed estimation technique is presented for constant speed and for a realistic speed profile as a function of time.

A. Mean Distance Between Local Minima of the Signal

The mean distance between the adjacent local minima of $g(x)$ is determined by the mean distance between adjacent positive-slope zero crossings of the derivative $g'(x)$. As given in Section III, the mean distance between adjacent positive-slope zero crossings of $g'(x)$ is $k_\lambda$. The constant $k$ is determined by the reciprocal of the zero crossing rate of $g'(x)$. The value of $k$ of 0.662 was calculated using the propagation model of Section II and numerical integration.

B. Selection of Significance Threshold

The threshold $\tau$ is selected such that the largest number of significant signed local extrema that are detected at scale $a_\tau$ is equal to the number of positive-slope zero crossings of the derivative $g'(x)$. Fig. 2 plots the ratio of the number of significant signed local extrema detected to the number of positive-slope zero crossings of $g'(x)$ as a function of the threshold $\tau$. The results shown here correspond to averages over 10 independent realizations of the Rayleigh fading and lognormal shadowing processes, each realization using approximately 1000 positive-slope zero crossings of $g'(x)$. Each of the four curves corresponds to a different analyzing wavelet $\psi(\tau)$. The wavelets chosen here are Daubechies’ filters of order 4 (db4) and 6 (db6), a “coiflet” of order 2 (coif1), and a symmetric, compactly supported biorthogonal spline wavelet (bior5.5) [11]. The optimum threshold for each wavelet is the value of $\tau$ for which the ordinate is 1.0 in Fig. 2. Table I summarizes some properties of the four analyzing wavelets and the corresponding optimum thresholds $\tau$. In addition to the wavelets mentioned above, one of the wavelets described in [13] was also used. This wavelet has two vanishing moments and is constructed using maximally flat, discrete-time lowpass and highpass filters with nine taps. The performance of this wavelet is similar to that of the coiflet (coif1).

The possibility of using the received envelope directly (instead of the logarithm of the envelope) was also investigated. One observation is that the local minima present in the envelope are not as prominent as those in the logarithm of the envelope. Furthermore, as the average received power $P(x,x)$ decreases, the magnitude of the CWT of the envelope also decreases, while the magnitude of the CWT of the logarithm of the envelope does not depend on the absolute received power. Since the optimum threshold $\tau$ would depend on the absolute received power if the CWT were applied directly to the received envelope, the logarithm of the envelope is chosen for signal analysis.

C. Effect of Number of Interarrival Times on Speed Estimate

Fig. 3 plots the normalized bias of the speed estimate, $E[\bar{v}/v] - 1$, as a function of the number of interarrival times (N) of adjacent signed local extrema for the four analyzing wavelets given in Table I. While the bias is negligible for large $N$, the value of $N$ must be sufficiently small in order to track

<table>
<thead>
<tr>
<th>Wavelet</th>
<th>Vanishing Moments</th>
<th>Length of Support</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>coif1</td>
<td>2</td>
<td>5</td>
<td>0.48</td>
</tr>
<tr>
<td>db4</td>
<td>4</td>
<td>7</td>
<td>1.18</td>
</tr>
<tr>
<td>bior5.5</td>
<td>6</td>
<td>9</td>
<td>0.78</td>
</tr>
<tr>
<td>db6</td>
<td>6</td>
<td>11</td>
<td>1.60</td>
</tr>
</tbody>
</table>

TABLE I
PROPERTIES OF ANALYZING WAVELETS AND VALUES OF THRESHOLD.
changes in mobile speed. Fig. 4 plots the normalized mean square error (MSE) of the speed estimate, $E[(1 - \psi)^2]$, as a function of number of interarrival times. For a given number of interarrival times, the coiflet (coif1) performs better than the other wavelets. As mentioned in Section III, the coiflet performs well since it has exactly two vanishing moments and hence minimizes the number of extraneous signed local extrema.

D. Choice of Maximum Scale, Duration of Observation Window, and Boundary Effects

The CWT is applied to blocks of the signal collected over time periods of duration $T_{\text{obs}}$ in order to limit the delay in obtaining speed estimates for real-time implementation. The determination of $T_{\text{obs}}$ is described in the following. Let the lowest speed of interest be denoted by $v_{\text{min}}$, i.e., speeds less than $v_{\text{min}}$ will be approximated by zero. If $L_{\psi}$ denotes the duration of support of the analyzing wavelet $\psi(t)$, the maximum scale for the CWT is

$$a_{\text{max}} = \frac{k\lambda}{L_{\psi}v_{\text{min}}},$$

(18)

where $k$ is determined in Section IV-A. The duration of the observation window is then the duration of the coarsest wavelet, i.e., $T_{\text{obs}} = L_{\psi}a_{\text{max}}$. The finite observation window results in boundary effects in the detection of the signed local extrema. In order to remove these effects, the observation windows are enlarged by $T_{\text{obs}}/2$ for each boundary (left and right endpoints). The enlarged windows slide by an amount $T_{\text{obs}}$ as time progresses. For the initial left boundary and the final right boundary, the signal $f(t)$ is reflected about the boundaries such that $f(t)$ and the first derivative $f'(t)$ are continuous at the boundaries. This technique of overlapping windows results in a factor of two increase in computation for the CWT and a $T_{\text{obs}}/2$ increase in delay of the speed estimate.

E. Application of Speed Estimation Technique

In this subsection, the speed estimation method is applied to specific functions of mobile speed versus time. Fig. 5 gives the estimated speed using various numbers of interarrival times $N$ for a constant mobile speed of 30 km/hr. The relevant parameters for this example which uses the coif1 wavelet are $v_{\text{min}} = 0.72$ km/hr, $\tau = 0.48$, and $M = 6$. The carrier wavelength is $\lambda = 1/3$ m, the correlation length of the lognormal shadowing is $\sigma_0(x) = 50$ m, and the exponent of distance dependence is $\alpha = 4$. The speed estimates were smoothed using a moving average of duration $T_{\text{obs}}$.

In contrast to the level crossing and covariance methods given in [1], speed estimation using wavelets as described here does not require the design of temporal windows which must adapt sufficiently quickly to track the variation of mobile speed while being robust to the variability in the statistics used for
speed estimation. Furthermore, since level crossing and covariance methods require estimates of the average received power, these speed estimators experience abrupt changes when the mobile makes a turn at a street corner. Since the method based on wavelets presented here does not require estimates of the average received power, there would be no sudden change in estimated speed at a street corner.

In order to demonstrate the tracking of changes in mobile speed, the speed estimator based on wavelets is applied to the following speed profile \( v(t) \) (in km/hr):

\[
v(t) = \begin{cases} 
0 & 0 \leq t < 0.5 \\
100 \gamma \left( \frac{t-0.5}{8} \right) & 0.5 \leq t < 8.5 \\
100 & 8.5 \leq t < 11.5 \\
100 \gamma \left( \frac{19.5-t}{8} \right) & 11.5 \leq t < 19.5 \\
0 & 19.5 \leq t \leq 20 \end{cases}
\]  

(19)

where

\[
\gamma(t) = \begin{cases} 
0 & t < 0 \\
3t^2 - 2t^3 & 0 \leq t < 1 \\
1 & 1 \leq t 
\end{cases}
\]  

(20)

Fig. 6 plots the actual speed \( v(t) \) given in (19) and the estimated \( \hat{v}(t) \). For this example, the parameters are the same as those used in Fig. 5 except \( v_{\text{min}} = 1.8 \text{ km/hr} \) and \( N = 20 \) (i.e., a maximum of \( N+1 = 21 \) nulls are used in the speed estimate; the actual number of nulls used in a neighborhood of time \( t \) depends on the speed \( v(t) \) and the duration of the observation window \( T_{\text{obs}} \)).

A fundamental quantity in evaluating the performance of the estimator is the normalized mean square error as a function of the number of interarrival times \( N \) (Fig. 4). While the absolute mean square error of the speed estimate depends on the mobile speed, the normalized mean square error is fixed for a given \( N \). This behavior is important for channel assignment and handoff algorithms whose performances depend on the spatial variation of the signal. It would be useful to compare this estimator with other techniques; however, other papers have not given algorithms for estimating variable mobile speeds. As shown in Fig. 6, the speed estimator presented here is able to track the changes in the mobile speed well.

V. CONCLUSION

This paper presents a new technique for estimating mobile speed in a Rayleigh fading environment. A wireless propagation model is proposed which accounts for correlated Rayleigh fading, correlated lognormal shadowing, and a distance-dependent trend. The characteristic spatial scale of the signal is mapped into a characteristic temporal scale through the mobile speed. With an empirically determined significance threshold, the significant signal local extrema of the continuous wavelet transform are used to detect the local minima in the logarithm of the received envelope. The interarrival times of adjacent significant local extrema are used to obtain an estimate of the mobile speed. Computer simulations indicate that the estimate is able to track a realistic, variable speed profile.

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