A Generalized Doppler Power Spectrum for Wireless Environments

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***Abstract*— We provide a direct derivation of a generalized Doppler power spectrum of the signal received by a mobile station traveling at a constant velocity. The derivation uses the three-dimensional (3-D) incident radiation intensity and the receiving antenna pattern to obtain an expression for the generalized power spectrum which is useful in environments where the 3-D propagation characteristics of wireless signals must be taken into consideration. A special case of radiation confined to a plane is considered to obtain the power spectrum givenin the book written by Jakes.**

***Index Terms*—Doppler spectrum, power spectrum, mobile communications, wireless communications.**

I. Introduction

We consider a mobile station traveling at a velocity v_z along the z-axis (Fig. 1). The received signal consists of many waves with different angles of arrival. Due to the velocity of the mobile station, each incoming wave will experience a frequency change (Doppler shift) which depends on the angle of arrival of the wave. A previous analysis of three-dimensional (3-D) propagation is given in [2] where the power spectrum is derived by first determining the autocorrelation of the received signal. Reference [3] applies the analysis of [2] to a specific form of the incident radiation intensity. Here, we present a direct derivation of a generalized Doppler power spectrum as a function of the 3-D incident radiation intensity and the receiving antenna pattern.

II. Notation

We denote p to be the average power received by an isotropic antenna. Using the coordinate system defined in Fig. 1, we let $pK_t(\theta, \phi)$ represent the incoming radiation intensity, i.e., power per unit solid angle. The gain of the receiving antenna is represented by $G(\theta, \phi)$. From these definitions, we have

$$\int K_t(\theta, \phi) d\Omega = 1 \tag{1}$$

$$\frac{1}{4\pi} \int G(\theta, \phi) d\Omega = 1 \tag{2}$$

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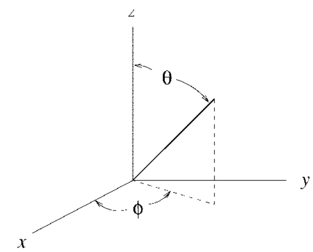

Fig. 1. Spherical coordinate system.

where Ω denotes solid angle. The average power received by the mobile station is [4]

$$W_r = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} pK_t(\theta, \phi)\, G(\theta, \phi) \sin\theta d\phi d\theta. \tag{3}$$

We define $h(\theta)$ as the integral with respect to ϕ in (3)

$$h(\theta) = \int_0^{2\pi} K_t(\theta, \phi) G(\theta, \phi) d\phi. \tag{4}$$

The function $h(\theta)$ contains the variation of both the receiving antenna pattern and the incoming radiation intensity with respect to the polar angle θ.

III. Generalized Power Spectrum

The generalized power spectrum is obtained by relating the Doppler frequency with the polar angle θ. A given Doppler frequency defines a cone at angle θ with the axis of symmetry in the direction of motion (z-axis). The relation between frequency f and angle θ is

$$f(\theta) = f_c + f_m \cos\theta \tag{5}$$

where f_c is the carrier frequency, $f_m = f_c v_z / c$ is the maximum Doppler frequency, and c is the speed of light. After a change of variable from angle θ to frequency f, (3) becomes

$$W_r = \frac{p}{f_m} \int_{f_c - f_m}^{f_c + f_m} h(\theta(f)) df \tag{6}$$

where $\theta(f) = \cos^{-1}((f - f_c)/f_m)$. The one-sided power spectrum $S(f)$ is defined such that

$$W_r = \int_0^{\infty} S(f) df. \tag{7}$$

Thus, the generalized one-sided Doppler power spectrum is given by

$$S(f) = \begin{cases} \dfrac{p}{f_m} h(\theta(f)), & |f - f_c| < f_m \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

The generalized power spectrum given by (8) is useful in environments such as the business centers of large cities where tall buildings demand consideration of the three-dimensional radiation intensity. By substituting an appropriate expression for $h(\theta)$, one can obtain a specific power spectrum for a particular environment and a given set of system parameters.

IV. Special Case

We now consider a special case with radiation confined to the xz-plane, as is done in [1]. Thus, we have

$$K_t(\theta,\phi) = K_1(\theta,0)\delta(\phi) + K_2(\theta,\pi)\delta(\phi-\pi) \tag{9}$$

where $\delta(\cdot)$ denotes the Dirac delta function. Substituting (9) into (4), we obtain

$$h(\theta) = G(\theta,0)K_1(\theta,0) + G(\theta,\pi)K_2(\theta,\pi). \tag{10}$$

Since the incoming radiation is restricted to the xz-plane, we define $ps(\theta)$ to be the radiation intensity per unit angle in the xz-plane. Thus,

$$s(\theta) = K_1(\theta,0)\sin\theta \tag{11}$$

$$s(-\theta) = K_2(\theta,\pi)\sin\theta \tag{12}$$

where $0 \le \theta \le \pi$. Using this definition and (5), (8), and (10), we obtain the power spectrum for this problem:

$$S(f) = \begin{cases} \dfrac{p[G(\theta(f),0)s(\theta(f)) + G(\theta(f),\pi)s(-\theta(f))]}{f_m\sqrt{1-\left(\dfrac{f-f_c}{f_m}\right)^2}}, & |f-f_c|<f_m \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Equation (13) agrees with the expression obtained in [1].

Carrying the special case one step further, we consider a mobile station which has an omnidirectional antenna with constant gain in the xz-plane (e.g., a half-wave dipole oriented along the y-axis). Then $G(\theta,0) = G(\theta,\pi) = G$. Also let the radiation intensity be uniform in the xz-plane, i.e., $s(\theta) = s(-\theta) = 1/(2\pi)$ for $0 \le \theta \le \pi$. This condition implies that $K_1(\theta,0) = K_2(\theta,\pi) = 1/(2\pi\sin\theta)$ for $0 \le \theta \le \pi$. The power spectrum now simplifies to

$$\frac{S(f)}{2pG/\omega_m} = \begin{cases} \dfrac{1}{\sqrt{1-\left(\dfrac{f-f_c}{f_m}\right)^2}}, & |f-f_c|<f_m \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

where $\omega_m = 2\pi f_m$. The power spectrum of (14) is plotted in Fig. 2.

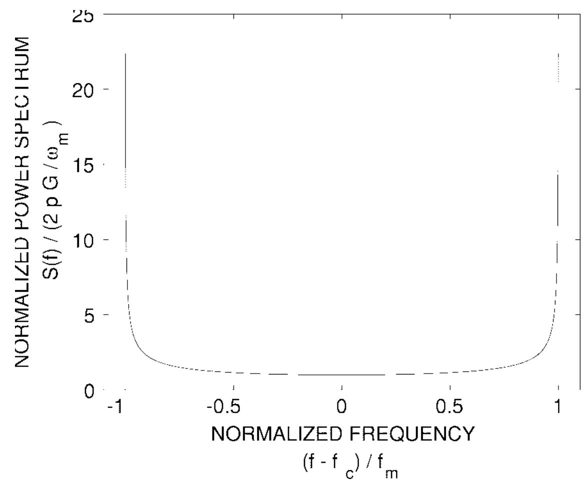

Fig. 2. Power spectrum for an omnidirectional antenna and for radiation intensity uniformly distributed in a plane.

V. Conclusion

The letter describes a generalized Doppler power spectrum which can be applied to environments where the 3-D propagation characteristics of wireless signals must be taken into account. The power spectrum given in [1] is shown to be a special case of the generalized power spectrum derived here.

References

[1] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.

[2] T. Aulin, "A modified model for the fading signal at a mobile radio channel," *IEEE Trans. Veh. Technol.*, vol. VT-28, pp. 182–203, Aug. 1979.

[3] J. D. Parsons and A. M. D. Turkmani, "Characterization of mobile radio signals: model description," *Proc. Inst. Elect. Eng.—I*, vol. 138, no. 6, pp. 549–556, Dec. 1991.

[4] S. Ramo, J. R. Whinnery, and T. van Duzer, *Fields and Waves in Communication Electronics*, 3rd ed. New York: Wiley, 1994.