Correction to “Multiple-Antenna Signal Constellations for Fading Channels”

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Abstract—The authors correct a mathematical equation contained in the correspondence “Multiple-antenna signal constellations for fading channels,” previously published in the IEEE TRANSACTIONS ON INFORMATION THEORY.

Index Terms—Multiple antennas, packings in Grassmannian space, Rayleigh flat fading.

I. INTRODUCTION

Unitary space-time constellations are useful in multiple-antenna systems operating in an environment where channel estimation is difficult. In [1], designing these unitary space–time constellations was related to the problem of finding a packing with a large minimum distance in the complex Grassmannian space. The paper proposed a design methodology and provided numerical results attained through the proposed design.

During subsequent work making use of this research, an error was found in one of the stated equations. This error affects the implementation of the methods given in [1]. We correct that error here.

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II. CORRECTION OF ERROR

In [1, eq. (13)], repeated here as (1), we have the unitary matrix

\[ U_{jk}^{p,q} (\phi_{pq}, \sigma_{pq}) = \begin{cases} 
1, & \text{if } j = k \text{ and } j \neq p, q \\
\cos(\phi_{pq}), & \text{if } j = k \text{ and } j = p, q \\
-\sin(\phi_{pq}) \exp(-i\sigma_{pq}), & \text{if } j = p \text{ and } k = q \\
\sin(\phi_{pq}) \exp(i\sigma_{pq}), & \text{if } j = q \text{ and } k = p \\
0, & \text{otherwise.} 
\end{cases} \]  

To illustrate one such \( U_{jk}^{p,q} \) we have

\[ U_{13}^{13} (\phi_{pq}, \sigma_{pq}) = \begin{pmatrix} 
\cos(\phi) & 0 & -\sin(\phi) \exp(-i\sigma) & 0 \\
0 & 1 & 0 & 0 \\
\sin(\phi) \exp(i\sigma) & 0 & \cos(\phi) & 0 \\
0 & 0 & 0 & 1 
\end{pmatrix}. \]

In [1, eq. (13)], repeated here as (2), we take a partial derivative of this unitary matrix

\[ \frac{\partial U(\Theta)}{\partial \phi_{pq}} \bigg|_{\Theta = 0} = U_{jk}^{p,q} \left( \frac{\pi}{2}, 0 \right). \]

This equation was stated erroneously. The corrected equation is

\[ \frac{\partial U(\Theta)}{\partial \phi_{pq}} \bigg|_{\Theta = 0} = \begin{cases} 
-1, & \text{if } j = p \text{ and } k = q \\
1, & \text{if } j = q \text{ and } k = p \\
0, & \text{otherwise.} 
\end{cases} \]  

To illustrate one such \( \frac{\partial U(\Theta)}{\partial \phi_{pq}} \) we have

\[ \frac{\partial U(\Theta)}{\partial \phi_{13}} \bigg|_{\Theta = 0} = \begin{pmatrix} 
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 
\end{pmatrix}. \]

Equation (3) should replace [1, eq. (13)].

The numeric results of [1] are still valid. We did an inspection of the software used to produce those results. Equation (3) stated here was used in that software.

REFERENCES